International Journal of Theoretical Physics, Vol. 38, No. 7, 1999

Cosmological Models with Matter Creation in Open Thermodynamic Systems

V. B. Johri[1,3] and Sanjay K. Pandey[2]

Received December 4, 1998

Regarding the universe as an open thermodynamic system, the creation of matter/radiation particles out of gravitational energy is investigated. A new class of FRW models with creation of matter is obtained and their properties are examined. A suitable choice of the particle number density function $n(t) = (A/t^{\alpha})^{3/2}$ leads to inflationary solutions during the particle creation phase; subsequently the universe enters the Friedmann era. It is found that for a physically acceptable solution $\alpha > 1$. A comparative study is made for $\alpha = 4/3$, 2, 8/3, and 10/3 in order to find a viable model of the universe.

1. INTRODUCTION

The cosmological observations of the hierarchy of galactic structure, intergalactic gases, dark matter, and cosmic microwave radiation inevitably lead to the question of the genesis of matter and radiation in the early universe. Since the existence of matter is a stark reality, there must be a plausible theory of its genesis to discuss the physics of creation in the universe; in fact the universal laws of nature remain incomplete without the incorporation of a creation mechanism in cosmology. In the absence of any plausible theory of creation of matter, some cosmologists just assume that all the matter, radiation, and even the high-energy vacuum emerged out of the big bang singularity where the known laws of physics including the mass-energy conservation laws break down. Such ad hoc assumptions about the instantaneous creation of matter from the singularity or its existence prior to the big bang do not make any scientific contribution.

[1] Department of Mathematics and Astronomy, Lucknow University, Lucknow 226007, India; e-mail: vinodjohri@hotmail.com.
[2] Department of Mathematics, L.B.S.P.G. College, Gonda 271001, India.
[3] To whom correspondence should be addressed.

To obviate the primordial singularity problem in the standard cosmological model, Hoyle and Narlikar[1] introduced the idea of the continuous creation of matter by incorporating a massless scalar field C in the Hilbert action. The resulting equations lead to the steady-state model of the universe. In this theory, the 4-momentum of the created particles is balanced by the 4-momentum of the C-field. As such, the creation C-field has negative energy, which induces matter creation and expansion of the universe. Although the C-field was introduced primarily to account for the continuous creation of matter, Narlikar[2] showed in 1973 that it also describes the explosive creation of matter such as required in big bang cosmology. Since the concept of the 'steady-state cosmology' was found to be incompatible with the observations of the evolution of the universe as confirmed by the cosmic microwave background radiation, Hoyle *et al.*[3] came out with quasi-steady state cosmology in 1993 according to which the C-field bosons pick up the threshold energy for matter creation only in the vicinity of massive objects; as such, creation takes place in the form of mini-big-bangs in the presence of strong gravitational fields.

Parker and co-workers[4] considered quantum mechanical particle creation (both matter and radiation) in the curved space-time of general relativity in the expanding universe. But significant particle creation by such a mechanism can take place only at very early times when the universe is expanding rapidly; it is also constrained by the expected backreaction effects, and as such it is not clear whether it can reasonably account for sufficient particle production to explain either the cosmic background radiation or the observable matter content of the universe. Later Brout and his collaborators[5] discussed the quantum creation of massive particles extracting energy from the background gravitational field, but the semiclassical equations used by them are adiabatic and time-symmetric and so they cannot account for the entropy production in the universe.

To obviate this problem, Prigogine *et al.*[6] followed a thermodynamic approach. Regarding the universe as an open thermodynamic system, they showed that the second law of thermodynamics may be modified to accommodate flow of energy from the gravitational field to the matter field, resulting in the creation of material particles. This leads to the reinterpretation of the stress tensor in general relativity, which now involves a time-asymmetric term depending on the rate of creation of particles. In other words, the process of particle creation out of gravitational energy is basically an irreversible phenomenon, capable of explaining the entropy burst in the expanding universe. Johri and Desikan[7] extensively used the thermodynamic approach to cosmology in investigating the creation of particles in a wide range of FRW models in the framework of both general relativity and the Brans–Dicke theory. Recently Lima, Waga, and collaborators[8] have given an interesting and critical account of adiabatic particle creation in FRW-type cosmologies. They have also discussed the relevance of the particle-creation cosmologies to solve the cosmic age problem as suggested by the recent direct measurements[9] of the Hubble constant.

One inherent problem in most particle creation cosmologies is that the behavior of the particle number function $N(t)$ or the variation of the particle number density $n(t)$ is left arbitrary, the system of the field equations being underdetermined since with the introduction of the particle creation mechanism there are more unknown variables involved than the number of independent equations. In fact, the functional form of $N(t)$ should be known from a fundamental theory of quantum processes during the nascent state of the universe. In the absence of any established theory of quantum gravity, the functional form of $N(t)$ or $n(t)$ has to be tentatively assumed in compatibility with the cosmological observations. Johri and coworkers[7] have already investigated models in this category. The present communication reports our recent findings in this connection by choosing a power-law form for the time dependence of $n(t)$.

Another important question linked with the Prigogine-type cosmologies is why there is matter creation out of gravitational energy, whereas other cosmologists have discussed matter creation out of the C-field,[1,2] high-energy vacuum in the early universe,[10] and bulk viscosity. The clue[11] to this question lies in a seminal result given by Landau and Lifshitz,[12] supported by Zeldovich,[13] and discussed analytically by Cooperstock,[14] Rosen,[15] and Johri *et al.*,[16] which states that *the total energy (material + gravitational) of the closed universe is zero*. This implies that any increase in the material energy of the universe must be accompanied by a corresponding decrease in the gravitational energy; in other words, matter creation must take place at the cost of gravitational energy.

Bulk viscosity is another irreversible phenomenon which contributes to entropy production in the universe. Padmanabhan and Chitre,[17] Johri and Sudharsan,[18] and Calvao *et al.*[19] have discussed the role of the bulk viscosity in the entropy production in an expanding universe. Johri and Sudharsan[18] have shown that the effect of the bulk viscosity is to reduce the thermodynamic pressure; in this sense it plays the role of negative isotropic pressure and acts like a creation field analogous to the vacuum energy field $\Lambda(t)$; however, all such models are characterized by the particle number conservation, in contrast to Prigogine-type cosmologies wherein the particle number $N(t)$ is time dependent and plays a pivotal role in the theory. Recently Sudharsan and Johri[20] have discussed the effect of bulk viscosity on the cosmological evolution of open thermodynamic systems which allow for simultaneous particle creation and entropy production; their investigations reveal that the production of specific entropy (entropy per particle) is independent of the nature of the creation characteristic function $\psi(t) \equiv \dot{N}/V$ and depends only upon the coefficient of the bulk viscosity. This suggests that bulk viscosity and particle creation are not only independent processes, but in general they lead to different histories of cosmic evolution.

The plan of the paper is as follows. In Section 2, we describe in brief the thermodynamic approach to particle creation as suggested by Prigogine and coauthors. Section 3 investigates the dynamical behavior of the field equations. We propose a new class of cosmological model for matter creation in Section 4. Section 5 describes the conclusions derived from our investigations and their implications.

2. THERMODYNAMIC APPROACH TO CREATION OF PARTICLES

Conventionally the universe is regarded as a closed thermodynamic system in which the conservation of the energy-momentum tensor alone is taken into account ignoring its possible interaction with the gravitational field. In order to introduce the idea of the creation of matter out of gravitational energy, Prigogine regards the universe as an open thermodynamic system in which the energy from the gravitational field may transform into the matter field. In the framework of cosmology, this approach allows for both particle and entropy production in the universe.

The cosmological model put forth by Prigogine *et al.*[6] takes the second law of thermodynamics into account from the beginning, leading to an extension of classical cosmology by a suitable reinterpretation of the material energy tensor in the Einstein field equations. Since the particles in this model are created at the expense of gravitational field energy, it involves an irreversible flow of energy from the gravitational field to the matter field.

If the universe were a closed system, then the second law of thermodynamics would have the usual form

$$dU = T\,dS - p\,dV \tag{1}$$

where U, T, S, p, and V denote internal energy, temperature, entropy, thermodynamic pressure, and volume, respectively. However, if we regard the universe as an open thermodynamic system which allows the entry of outside particles, Eq. (1) takes the form

$$dU = T\,dS - p\,dV + \mu\,dN \tag{2}$$

where N is the number of particles at any instant of time in a given volume V and $\mu = [\partial U/\partial N]_{S,V}$ defines the chemical potential, i.e., the change of internal energy per particle added to the system at a constant entropy S and volume V.

In such a transformation, the thermal energy received by the system is due entirely to the change of the number of particles, and the total number of particles $N(t)$ is no longer a constant, but is time dependent. In cosmological context, this change is due to the transfer of energy from the gravitational field to the matter field. Hence the creation of matter acts as a source of internal energy.

Now S is an extensive property of the system, i.e., S is proportional to the number of particles included in the system, say $S = \sigma(t)N$, where $\sigma(t)$ is the specific entropy. It has been shown (see Lima[8]) that during the adiabatic particle creation scenario, $\sigma(t)$ remains time independent. It follows that for adiabatic particle creation in the homogeneous cosmological models

$$\frac{dS}{S} = \frac{dN}{N} \tag{3}$$

Therefore the increase in the entropy dS is only due to the creation of particles. Combining Eqs. (2) and (3), we get

$$dU = TS\left(\frac{dN}{N}\right) - p\, dV + \mu\, dN \tag{4}$$

Since U and V are also extensive properties of the system, we have

$$\frac{dU}{U} = \frac{dN}{N} \quad \text{and} \quad \frac{dV}{V} = \frac{dN}{N}$$

Inserting these values in (4), we obtain

$$\mu N = U + pV - TS = \tilde{H} - TS \tag{5}$$

where $\tilde{H} = U + pV$ is the enthalpy of the system.

Substituting the value of μ from Eq. (5) into Eq. (4), we get

$$dU = -p\, dV + \frac{\tilde{H}}{N}\, dN \tag{6}$$

or

$$d(\rho V) = -(p + p_c)\, dV \tag{7}$$

where ρ is the energy density of the system and

$$p_c = -\frac{\tilde{H}}{N}\frac{dN}{dV} \tag{8}$$

Equation (7) suggests that the creation of matter in an open thermodynamic system is equivalent to adding a term p_c given by Eq. (8) to the thermodynamic pressure p.

Again, the second law of thermodynamics demands that

$$dS \geq 0$$

Therefore using Eq. (3), the above condition reads

$$dN \geq 0 \tag{9}$$

This is an important result, as it clearly indicates that the creation of particles in the expanding universe is an irreversible process; in other words, gravitational energy can produce matter, but the reverse process is thermodynamically forbidden. This is consistent with the result obtained by Gunzig and Nardone[21] that a vacuum fluctuation in the primordial state is structurally unstable with respect to irreversible matter creation.

3. DYNAMICAL BEHAVIOR OF THE FIELD EQUATIONS

In the presence of matter creation, the energy-momentum tensor takes the form

$$\tilde{T}_{ik} = (p + \rho + p_c)u_i u_k - (p + p_c)g_{ik} \tag{10}$$

where u_i is the 4-velocity vector.

Accordingly, the field equations of general relativity have the form

$$R_{ik} - \tfrac{1}{2}Rg_{ik} = 8\pi G\tilde{T}_{ik} \tag{11}$$

For the standard FRW metric

$$ds^2 = dt^2 - R^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)\right]$$

Eq. (11) leads to

$$3\left[H^2 + \frac{k}{R^2}\right] = 8\pi G\rho \tag{12}$$

and

$$2\dot{H} + 3H^2 + \frac{k}{R^2} = -8\pi G(p + p_c) \tag{13}$$

where $H = \dot{R}/R$ is the Hubble constant.

Combining (12) and (13), we get the energy conservation equation as

$$\dot{\rho} + (p + \rho)3H = -3p_c H = (p + \rho)\frac{\dot{N}}{N} \tag{14}$$

Using the barotropic equation of state

$$p = \gamma p, \qquad 0 \leq \gamma \leq 1 \tag{15}$$

we find that Eq. (14) yields on integration

$$N(t) = N_0 R^3 \rho^{1/(1+\gamma)} \tag{16}$$

If V denotes the comoving volume and n the particle number density, we have

$$N = nV = nR^3 \tag{17}$$

Combining (16) and (17), we get

$$\rho = Kn^{1+\gamma} \tag{18}$$

where K is a proportionality constant.
Also for relativistic particles, we have

$$\rho = aT^4, \qquad \gamma = 1/3 \tag{19}$$

where a is the radiation constant.
Combining (18) and (19) we get

$$T \propto n^{1/3} \tag{20}$$

It is noteworthy that in the particle-creation scenario discussed above the dependence of energy density ρ and temperature T on the particle number density holds irrespective of the functional form of the particle number function $N(t)$. This marks the departure from the standard cosmology to a more general class of cosmologies with matter creation, but the system of field equations is now underdetermined as there are only three independent equations (12), (14), and (15) involving four unknowns. This makes the choice of $N(t)$ or $n(t)$ arbitrary and the reasonable course is to choose a functional form of $n(t)$ which is compatible with the cosmological observations.

4. THE COSMOLOGICAL MODEL

In the very early universe, the spatial curvature term k/R^2 may be neglected and Eq. (12) yields

$$\rho = \frac{3H^2}{8\pi G} \tag{21}$$

In conjunction with (18), this yields for relativistic particle ($\gamma = 1/3$)

$$Kn^{4/3} = \frac{3H^2}{8\pi G} \tag{22}$$

It relates the particle number density n to the Hubble constant H. The field equations in Section 3 now reduce to the single equation (22) and we cannot proceed further unless n is known. An exact form of such a function should be determinable from quantum field theory in the presence of gravitation. However, in the absence of a plausible theory of quantum gravity, the natural way is to investigate a physically interesting form of $n(t)$as discussed below.

It is obvious from Eq. (22) that during the particle creation scenario, the particle number density n depends implicitly on the cosmic time t (being proportional to $H^{3/2}$). This is very much analogous to the standard Friedmann model wherein $n \propto 1/t^2$ (matter-dominated universe) or $n \propto T^3 \propto 1/t^{3/2}$ (radiation-dominated universe). By analogy, we assume a suitable power law generalization of the above by taking

$$n = \left(\frac{A}{t^{\alpha}}\right)^{3/2} \tag{23}$$

during the particle-creation scenario, where A is a constant of proportionality; the exponent 3/2 is taken for mathematical simplification. Combination of (22) and (23) yields

$$R = R_1 e^{\xi t^{-\alpha+1}} \tag{24}$$

where

$$\xi = \left(\frac{8\pi GK}{3}\right)^{1/2} \frac{A}{\alpha - 1}$$

Therefore

$$\dot{R} = \xi(\alpha - 1)Rt^{-\alpha} \tag{25}$$

implying

$$H = \xi(\alpha - 1)t^{-\alpha}$$

In the expanding universe $H > 0$, therefore the above equation suggests that $\alpha > 1$

Also (25) gives

$$\ddot{R} = \xi(\alpha - 1)Rt^{-\alpha-1}[\xi(\alpha - 1)t^{-\alpha+1} - \alpha] \tag{26}$$

Also, from Eqs. (16) and (21)

$$\frac{\dot{N}}{N} = 3H + \frac{3\dot{H}}{2H} = \frac{3}{t}\left[\frac{\xi(\alpha - 1)}{t^{\alpha-1}} - \frac{\alpha}{2}\right] \tag{27}$$

We have the following observations from Eqs. (26) and (27)

1. The model shows accelerated expansion leading to inflation for $t < [\xi(1 - 1/\alpha)]^{1(\alpha-1'(}$.
2. The expansion rate starts decelerating for $t > [\xi(1 - 1/\alpha)]^{1/(\alpha-1)}$.
3. Creation stops at $t = [2\xi\,(1 - 1/\alpha)]^{1/(\alpha-1)}$; thereafter, as obvious from Eq. (14), the universe enters the Friedmann era represented by a radiation-dominated model.

We have from Eqs. (20) and (23)

$$T \propto \frac{1}{t^{\alpha/2}} \Leftrightarrow t \propto \frac{1}{T^{2/\alpha}} \tag{28}$$

Therefore,

$$\left(\frac{t_{\mathrm{GUT}}}{t_{\mathrm{Pl}}}\right) = \left(\frac{T_{\mathrm{Pl}}}{T_{\mathrm{GUT}}}\right)^{2/\alpha} = \left[\frac{10^{19}\ \mathrm{GeV}}{10^{14}\ \mathrm{GeV}}\right]^{2/\alpha} = (10^5)^{2/\alpha}$$

The GUT epoch in this model is therefore given by

$$t_{\mathrm{GUT}} = (10^5)^{2/\alpha} t_{\mathrm{Pl}}\ \mathrm{sec} = (10^5)^{2/\alpha} \cdot 10^{-43}\ \mathrm{sec} \tag{29}$$

This gives rise to a variety of FRW cosmologies with matter creation for different values of α, as given in Table I.

Again, we can determine the parameter ξ by the requirement that creation

Table I

α	Particle density	Temperature	GUT epoch in the model
4/3	$n \propto 1/t^2$	$T \propto 1/t^{2/3}$	$t_{\mathrm{GUT}} \sim 10^{-35.5}$ sec
2	$n \propto 1/t^3$	$T \propto 1/t$	$t_{\mathrm{GUT}} \sim 10^{-38}$ sec
8/3	$n \propto 1/t^4$	$T \propto 1/t^{4/3}$	$t_{\mathrm{GUT}} \sim 10^{-39}$ sec
10/3	$n \propto 1/t^5$	$T \propto 1/t^{5/3}$	$t_{\mathrm{GUT}} \sim 10^{-40}$ sec

must stop at the GUT epoch in order to obviate the magnetic monopole problem. This gives

$$\left[2\xi\left(1-\frac{1}{\alpha}\right)\right]^{1/(\alpha-1)} = (10^5)^{2/\alpha}\cdot 10^{-43}$$

5. CONCLUSION

Table I lists a variety of FRW-type cosmological models with particle creation. It is clear that as we increase the exponent α, the GUT epoch in the corresponding model approaches closer to the Planck epoch. The widely accepted value for the GUT phase transition to occur is around 10^{-36} sec, which is very close to our model for $\alpha = 4/3$. Now the order of inflation of the universe during the particle-creation phase starting from the Planck epoch can be obtained by using Eq. (24) as follows

For $\alpha = 4/3$

$$\frac{R_f}{R_t} = \exp[-\xi(t_{\mathrm{GUT}}^{-1/3} - t_{\mathrm{Pl}}^{-1/3})] = e^{10^2}$$

where R_i and R_f denote the scale factor when particle creation starts and stops, respectively. Accordingly

$$\frac{R_f}{R_i} \approx 10^{43}$$

which is more than enough to solve the horizon and flatness problems inherent in the standard model.

ACKNOWLEDGMENTS

This work was done under the CSIR Emeritus Scientist Project at Lucknow University.

REFERENCES

1. F. Hoyle and J. V. Narlikar, *Proc. Roy. Soc. A* **270**, 334 (1962).
2. J. V. Narlikar, *Nature* **242**, 135 (1973).
3. F. Hoyle, G. Burbidge, and J. V. Narlikar, *Ap. J.* **410**, 437 (1993); *Mon. Not. R. Astron. Soc.* **267**, 1007 (1994); *Astron. Astrophys.* **289**, 729 (1994).
4. L. Parker, *Phys. Rev. Lett.* **21**, 562 (1968); *Phys. Rev.* **183**, 1057 (1969); S. A. Fulling, L. Parker, and B. L. Hu, *Phys. Rev. D* **10**, 3905 (1974); B. L. Hu and L. Parker, *Phys. Rev. D* **17**, 933 (1978).
5. R. Brout, F. Englert, and E Gunzig, *Ann. Phys.* (N.Y.) **115**, 78 (1979); *Nucl. Phys. B* **170**, 228 (1980).

6. I. Prigogine and J. Geheniau, *Proc. Natl. Acad. Sci. USA* **83**, 6245 (1986); I. Prigogine, J. Geheniau, E. Gunzig, and P. Nardone, *Proc. Natl. Acad. Sci. USA* **85**, 7428 (1988); *Gen. Rel. Grav.* **21**, 767 (1989).
7. V. B. Johri and K. Desikan, *J. Math. Phys. Sci.* **25**, 563 (1991); *J. Ind. Math. Soc.* **61**, 42, (1995); *Astron. Lett. Commun.* **33**, 287 (1996); V. B. Johri, IUCAA preprint no. 02/97 (1997).
8. J. A. S. Lima and A. S. M. Germano, *Phys. Lett. A* **170**, 373 (1992); J. A. S. Lima, A. S. M. Germano, and L. R. W. Abramo, *Phys. Rev. D* **53**, 4287 (1996); J. A. S. Lima, *Phys. Rev. D* **54**, 2571 (1996).
9. M. J. Pierce, D. L. Welch, R. D. McClure, S. Van den Bergh, R. Racine, and P. B. Stetson, *Nature* **371**, 29 (1994); W. L. Freedman *et al.*, *Nature* **371**, 27 (1994).
10. A. H. Guth, *Phys. Rev.* **23**, 347 (1981); A. D. Linde, *Phys. Lett.* **108B**, 389 (1982); *Phys. Lett.* **129B**, 177 (1983); C. Mathiazhagan and V. B. Johri, *Class. Quantum Gravity* **1**, L29 (1984); M. Ozer and M. O. Taha, *Nucl. Phys. B* **287**, 776 (1987); K. Freese, F. C. Adams, *et al.*, *Nucl. Phys. B* **287**, 797 (1987).
11. A. H. Guth, *The Oscar Klein Memorial Lectures*, Vol. 2, World Scientific, Singapore (1991); S. W. Hawking, *A Brief History of Time*, Bantam, New York (1988); V. B. Johri, IUCAA, Annual Report (1996–97), p. 51.
12. L. D. Landau and E. F. Lifshitz, *The Classical Theory of Fields*, Pergamon, Oxford.
13. Ya. B. Zeldovich, *Sov. Phys. JETP* **16**, 732 (1963).
14. F. I. Cooperstock, *Found. Phys.* **22**, 1011 (1992); *Gen. Rel. Grav.* **26**, 323 (1994).
15. N. Rosen, *Gen. Rel. Grav.* **26**, 319 (1994).
16. V. B. Johri, D. Kalligas, G. P. Singh, and C. W. F. Everitt, *Proceedings Second William Fairbank Conference*, World Scientific, Singapore (1993); *Gen. Rel. Grav.* **27**, 313 (1995).
17. T. Padmanabhan and S. M. Chitre, *Phys. Lett. A* **120**, 433 (1987).
18. V. B. Johri and R. Sudharsan, *Phys. Lett. A* **136**, 316 (1988).
19. M. O. Calvao, J. A. S. Lima, and I. Waga, *Phys. Lett. A* **162**, 233 (1992).
20. R. Sudharsan and V. B. Johri, *Gen. Rel. Grav.* **26**, 41 (1994).
21. E. Gunzig and P. Nardone, *Gen. Rel. Grav.* **16**, 305 (1984).